It should be noted that the experiment produced relatively high tunnel blockage (16% and the experimental data given in Fig. 3 have been corrected as suggested by Pope,[3] to approximate a body in a freestream.

Conclusions

The moving tuft procedure for locating a stagnation point on a body in a two-dimensional flow has been demonstrated to be a simple and reliable experimental method. The method can be easily adapted to any blunt nosed body under conditions where the stagnation zone can be visually observed during the test. The good agreement noted between the moving tuft procedure and a more "standard" procedure warrants further use and critical examination of this new technique in the process of locating a stagnation point.

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Subsonic Loads on Wings having Sharp Leading Edges and Tips

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Introduction

IN an earlier paper,[1] the authors presented a vortex-lattice method which, for incompressible flows, accurately predicts the aerodynamic characteristics of wings having separation at the sharp edges. The method is reliable for any angle of attack as long as vortex bursting does not occur in the immediate vicinity of the wing. In the present paper, we extend this method to compressible subsonic flows using a modified Prandtl-Glauert transformation.

The Prandtl-Glauert transformation was used in conjunction with a vortex-lattice method in several earlier studies.[2-4] This transformation was also used in other attempts to treat wings having low aspect ratios.[5,6] For wings having aspect ratios up to 4.5, the wakes adjoining the wing tips for rectangular wings can strongly influence the flow over a significant portion of the wing, even for moderate angles of attack (6°-10°). Hence, the accurate prediction of all the aerodynamic coefficients depends on how well these wakes are modeled.[1] In the previous studies of compressible flows,[2-4] the positions of the wakes were prescribed, not obtained as part of the solution. Thus, the earlier methods are at best applicable at small angles of attack, though Ermolenko[4] did not make such an assumption in modifying the Prandtl-Glauert transformation.

We do not assume small angles of attack and, in a manner similar to Ermolenko's, modify the Prandtl-Glauert transformation. The present approach is valid for moderate local angles of attack if the freestream Mach number is low, and for high subsonic, freestream Mach numbers if the local angles of attack are small. In contrast with the previous studies, we do not prescribe the positions of the wakes; instead the wakes are treated as in the incompressible case.[1] Numerical results showing the effect of the freestream Mach number on the aerodynamic coefficients are compared with available experimental data for several planforms.

Transformations and Method of Solution

Neglecting the nonlinear terms in the disturbed velocity leads to the following nondimensional form of the governing equation

$$B^2 \frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial y'^2} + \frac{\partial^2 \phi}{\partial z'^2} = 0 \tag{1}$$

where ϕ is the disturbance potential and $B=(1-M_\infty^2)^{1/2}$. We introduced a coordinate system (x', y', z') such that x'-axis is parallel to the freestream. Equation (1) can be transformed into Laplace's equation

$$\frac{\partial^2 \phi}{\partial x_i'^2} + \frac{\partial^2 \phi}{\partial y_i'^2} + \frac{\partial^2 \phi}{\partial z_i'^2} = 0 \tag{2}$$

by introducing the Prandtl-Glauert transformation

$$x_i' = x'/B, \; y_i' = y', \text{ and } z_i' = z' \tag{3}$$

we refer to (x', y', z') as the "compressible space" and to (x_i', y_i', z_i') as the "incompressible space."

Next we introduce new coordinates (x, y, z) and (x_i, y_i, z_i) in both the compressible and incompressible spaces, respectively. The y-axis is perpendicular to the wing, and the y_i-axis is perpendicular to its image. The relationships needed to transform the "compressible problem" into an equivalent "incompressible problem" are derived in Ref. 7. These relationships include the geometrical parameters of the wing, the disturbance and total velocities, the distribution of circulation, the boundary conditions, the streamlines, and finally the distributed and total loads. The method of solution is briefly described as follows.

The wing in the compressible space is transformed into its image in the incompressible space. Then the incompressible problem is solved as described in Ref. 1 subject to the relationships of transformation of the velocities, the

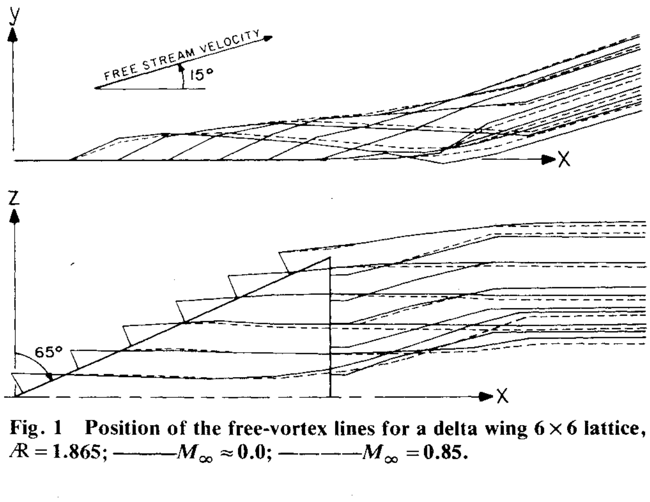

Fig. 1 Position of the free-vortex lines for a delta wing 6×6 lattice, Æ = 1.865; ———$M_\infty \approx 0.0$; — — —$M_\infty = 0.85$.

Received July 17, 1975; revision received October 3, 1975. This work was supported by the NASA Langley Research Center under Grant NGR 74-004-090.

Index category: Aircraft Aerodynamics (including Component Aerodynamics).

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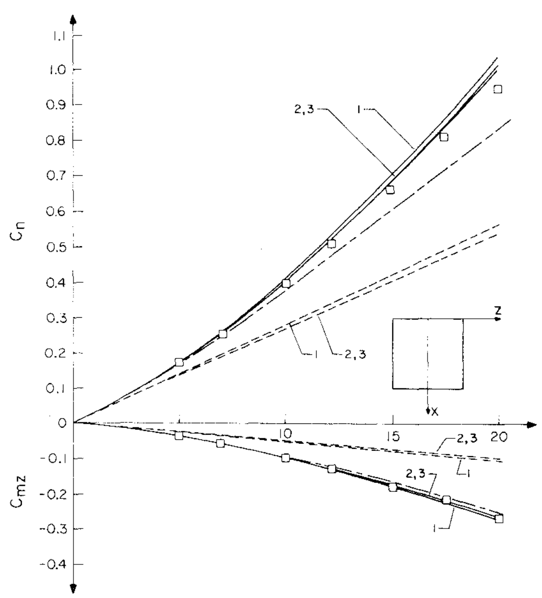

Fig. 2 Variations of the normal-force coefficient and pitching moment-coefficient vs angle of attack $M_\infty = 0.5$, $Æ = 1$. Numerical results: ——Ermolenko[4]; present method, ———linear; ———nonlinear. Curve no. 1 = 4 × 4 lattice; Curve no. 2 = 6 × 6 lattice; and Curve no. 3 = 9 × 9 lattice. Experimental results: □ Winter,[8] $M_\infty \simeq 0.1$.

boundary condition, and the streamlines.[7] The load coefficients for the transformed wing are calculated and the relationships for transforming the loads are used to obtain the corresponding coefficients for the actual wing. Finally, the shape of the wake for the actual wing is obtained using the geometrical relationships of transformation.

Numerical Examples

Here, we consider rectangular and delta wings. The wings are thin flat plates. For rectangular wings, separation is considered only at the wing tips, while for delta wings separation is considered at the leading edges. The edges where separation occurs are considered to be very sharp; i.e., the bevel angles of these edges is very small.

Figure 1 depicts an actual solution of a delta wing for freestream Mach numbers of 0.0 and 0.85 shown by the solid and dotted lines, respectively. It can be noted that the effect of compressibility on the wake tends to move the free-vortex lines nearer to the plane of symmetry of the wing and upward from the wing plane.

In Fig. 2, the normal-force and pitching-moment coefficients for a rectangular wing are given as functions of the angle of attack for $M_\infty = 0.5$. The curves based on the latices having six rows by six columns and nine rows by nine columns are practically identical. It can be noted that they are practically identical to the experimental results of Winter[8] ($M_\infty \simeq 0.1$) until the angle of attack reaches 15°. Above this angle, the normal-force coefficient is slightly larger than the experimental results. The numerical results of Ermolenko[4] ($M_\infty = 0.5$) underestimate the normal-force coefficient.

In Fig. 3, the same coefficients for a delta wing having an aspect ratio of two is given as functions of the angle of attack for various values of M_∞. The agreement with experimental data is good up to 12°. Above this angle of attack the present method overpredicts the load coefficients, especially for high values of M_∞.

Conclusions

The numerical examples show that the present method is successful in predicting the aerodynamic loads on low-aspect

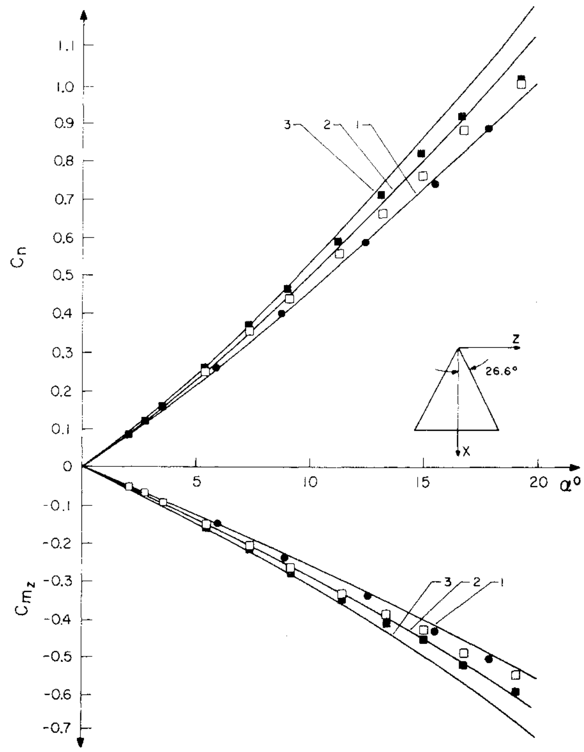

Fig. 3 Variations of normal-force and pitching moment coefficient with angle of attack $Æ = 2$. Numerical results: ———present method (6x6 lattice). Curve no. 1: $M_\infty = 0.0$. Curve no. 2: $M_\infty = 0.7$. Curve no. 3: $M_\infty = 0.85$. Experimental results: ● low M_∞, Bartlett and Vidal;[9] from Emerson,[10] □ $M_\infty = 0.7$, and ■ $M_\infty = 0.85$.

wings at moderate angles of attack for high subsonic M_∞. The method is limited to angles of attack up to 12° for high subsonic M_∞ and to angles of attack up to 20° for $M \leq 0.5$. For high subsonic M_∞ and large angles of attack a method such as that proposed in Ref. 7 may be used.

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